Review – Convolution 12/2/25

Discrete-Time Convolution of $h[n]$ and $x[n]$

$$y[n] = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$$

Continuous-Time Convolution of $h(t)$ and $x(t)$

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

Applications

- Responses of LTI systems in the time domain with impulse response h: $y = h * x$
- Correlation (slide instead of flip-and-slide)
- Random variables: $z = a + b$

$$\underbrace{f_Z(z)}_{\text{pdf}} = \underbrace{f_A(z)}_{\text{pdf}} * \underbrace{f_B(z)}_{\text{pdf}}$$

pdf = probability density function

only if R.V.'s ~~and~~ a and b are statistically independent

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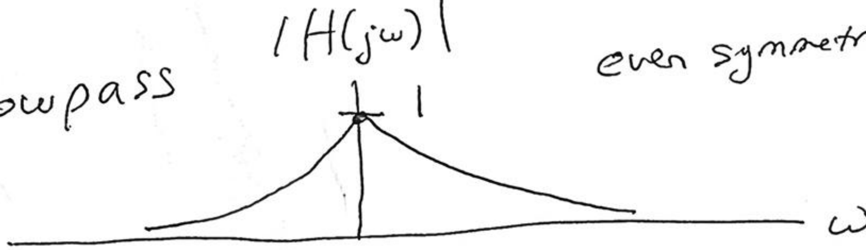
$$\left.\frac{2e^{(-2-j\omega)t}}{-2-j\omega}\right|_0^\infty =$$

$$\lim_{t\to\infty} 2e^{(-2-j\omega)t} - \frac{2}{-2-j\omega}$$

$$\lim_{t\to\infty} \underbrace{2\overset{\nearrow 0}{\cancel{e^{-2t}}}}_{\substack{\text{goes to}\\ \text{zero}\\ \text{as } t\to\infty}} \underbrace{e^{-j\omega t}}_{\text{oscillating}} + \frac{2}{2+j\omega} = \frac{2}{2+j\omega}$$

$$H(j\omega) = \frac{2}{2+j\omega} \rightsquigarrow \quad H(j0) = \frac{2}{2} = 1, \quad \lim_{\omega\to\infty} |H(j\omega)| = 0$$

Lowpass even symmetric



$$H(j\omega) = \frac{2}{2+j\omega} \cdot \frac{2-j\omega}{2-j\omega} = \frac{4-j2\omega}{\underset{4}{\cancel{2}} + j\cancel{2\omega} - j\cancel{2\omega} + \underbrace{\omega^2}_{-j^2\omega^2}} = \frac{4-j2\omega}{\underset{4}{\cancel{2}}+\omega^2}$$